

CLASS XII - VECTOR

1. If a and b are unit vectors, then what is the angle between a and b for $\sqrt{3}a - b$ to be a unit vector?
(a) 30° (b) 45° (c) 60° (d) 90°
2. The value of λ , for which vectors $2i + j + 3k$ and $i - \lambda j + 4k$ are orthogonal is _____.
3. Find the angle between the vectors $a = i - j + k$ and $b = i + j - k$.
4. If $|a| = \sqrt{3}$, $|b| = 2$ and angle between a and b is 60° , find $a \cdot b$.
5. Find the projection of a on b if $a \cdot b = 8$ and $b = 2i + 6j + 3k$.
6. If p is a unit vector and $(x - p) \cdot (x + p) = 80$, then find $|x|$.
7. If $a \cdot a = 0$ and $a \cdot b = 0$, then what can be concluded about the vector b ?
[Foreign 2011]
8. Find λ , when the projection of $a = \lambda i + j + 4k$ on $b = 2i + 6j + 3k$ is 4 units.
[Delhi 2012]
9. Show that $|a + b|^2 = |a|^2 + |b|^2$, if a and b are along adjacent sides of a rectangle.
10. If a is a unit vector and $(x - a) \cdot (x + a) = 8$, find $|x|$. [NCERT]
11. Find the magnitude of each of the two vectors a and b , having the same magnitude such that the angle between them is 60° and their scalar product $9/2$.
[CBSE 2018]
12. If two vectors a and b are such that $|a| = 2$, $|b| = 1$ and $a \cdot b = 1$, then find the value of $(3a - 5b) \cdot (2a + 7b)$.
[Delhi 2011]
13. Find the projection of $b + c$ on a , where
 $a = 2i - 2j + k$, $b = i + 2j - 2k$ and $c = 2i - j + 4k$.
14. If $a = 7i + j - 4k$ and $b = 2i + 6j + 3k$, then find the projection of a on b .
[Delhi 2015]
15. If $|a| = 2$, $|b| = \sqrt{3}$ and $a \cdot b = \sqrt{3}$, find the angle between a and b .
16. If a and b are two unit vectors and θ is the angle between them, then show that $1/2 (a - b)^2 = 1 - \cos \theta$.
17. If $(a + b) \cdot (a - b) = 12$ and $|a| = 2$, find $|a|$ and $|b|$.

❖ Long Answer I / Long Answer II Type

18. If $a = i + 2j - 3k$, $b = 3i - j + 2k$, show that $(a + b)$ and $(a - b)$ are perpendicular to each other.
19. The scalar product of the vector $i + j + k$ with the unit vector along the sum of vectors $2i + 4j - 5k$ and $\lambda i + 2j + 3k$ is equal to one. Find the value of λ .
20. If a, b are any two vectors, then give the geometrical interpretation of the relation $|a + b| = |a - b|$
21. Show that each of the given three vectors is a unit vector
 $1/7(2i + 3j + 6k)$, $1/7(3i - 6j + 2k)$, $1/7(6i + 2j - 3k)$.
Also, show that they are mutually perpendicular to each other.
[NCERT]
22. If the vertices A, B, C of a ΔABC have position vectors $(1, 2, 3)$, $(-1, 0, 0)$ and $(0, 1, 2)$ respectively, what is the magnitude of $\angle ABC$?
23. If $a+b+c = 0$ and $|a| = 3$, $|b| = 5$ and $|c| = 7$, then find the angle between a and b . [Delhi 2014]
24. If a, b, c are mutually perpendicular vectors of equal magnitudes, show that the vector $a+b+c$ is equally inclined to a, b and c . Also, find the angle which $a+b+c$ makes with a or b or c .
[Delhi 2017]
25. If $a = 2i - j - 2k$ and $b = 7i + 2j - 3k$, then express b in the form of $b = b_1 + b_2$, where b_1 is parallel to a and b_2 is perpendicular to a .
[AI 2017]
26. If vectors $a = 2i + 2j + 3k$, $b = -i + 2j + k$ and $c = 3i + j$ are such that $a + \lambda b$ is perpendicular to c , then find the value of λ .
[NCERT, Foreign 2011]
27. Dot product of a vector with vectors $i - j + k$, $2i + j - 3k$ and $i + j + k$ are respectively 4, 0 and 2. Find the vector. [Delhi 2013(C)]
29. If a and b are two vectors such that $|a + b| = |a|$, then prove that vector $2a + b$ is perpendicular to vector b .

❖ PRACTICE QUESTIONS**Very Short (Objective Type) / Short Answer Type**

1. For any non zero vector a ,
 $a \cdot (a \cdot i) + (a \cdot j) + (a \cdot k)$. State true or false.
2. If $|a + b| = |a - b|$, then a and b are perpendicular.
State true or false.
3. Write the projection of the vector $i - j$ on the vector $i + j$.
[NCERT, AI 2011]
4. If a is a unit vector and $(2x - 3a) \cdot (2x + 3a) = 91$, then write the value
of $|x|$. [Delhi 2013(C)]
5. Find the projection of the vector $i + 3j + 7k$ on the vector $2i - 3j + 6k$.
[Delhi 2014]
6. For what value of λ are the vectors $a = 2i + 2j + k$ and $b = -i - 2j + 3k$
perpendicular to each other? [Delhi 2013(C)]
7. If a and b are two unit vectors such that $a + b$ is also a unit vector,
then find the angle between a and b . [Delhi 2014]
8. If a, b and c are mutually perpendicular unit vectors, then find the
value of $|2a + b + c|$.
9. Find the length of the sum of the three mutually perpendicular unit
vectors.
10. If $(a + b)$ is perpendicular to b and a is perpendicular to $(2b + a)$ then
show that $a^2 = 2b^2$.

Long Answer I / Long Answer II Type

11. If $a = 3i + 4j + 5k$ and $b = 2i + j - 4k$, then express b in the form $b = b_1 + b_2$, where b_1 is parallel to a and b_2 is perpendicular to a .
[Foreign 2012]
12. If $a = i - j + 7k$ and $b = 5i - j + \lambda k$, then find the value of λ , so that $a + b$ and $a - b$ are perpendicular vectors.
[AI 2013]
13. If a, b and c are three mutually perpendicular vectors of equal
magnitude, prove that the angle which $(a + b + c)$ makes with any of
the vectors a, b or c is $\cos^{-1} (1 / \sqrt{3})$. [DSE]

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Result Oriented

(DAILY PRACTICE PAPER)

[CLASS XII]

14. If a and b are two unit vectors and θ is the angle between them, then show that $\sin(\theta/2) = (1/2)|a - b|$. [HOTS]
15. The scalar product of the vector $a - i + j + k$ with a unit vector along the sum of the vectors $b = 2i + 4j - 5k$ and $c = \lambda i + 2j + 3k$ is equal to one. Find the value of λ , and hence find the unit vector along $b + c$. [AI 2014]
16. If a, b, c are three vectors such that $|a| = 5, |b| = 12$ and $|c| = 13$, and $a + b + c = 0$, find the value of $a \cdot b + b \cdot c + c \cdot a$. [Delhi 2012]
17. Show that the angle between any two diagonals of a cube is $\cos^{-1}(1/3)$.

❖ CROSS PRODUCT OR VECTOR PRODUCT

1. Vector or cross product of two vectors a and b , denoted by $a \times b$, is given by $|a \times b| = |a||b| \sin \theta$, where θ is angle between a and b and \hat{n} is a unit vector perpendicular to a and b and direction is such that a, b and \hat{n} form a right hand system.
2. $|a \times b|$ is a vector quantity, whose magnitude is, $|a \times b| = |a||b| \sin \theta$.
3. If θ is angle between a and b , then $\sin \theta = |a \times b| / |a||b|$
4. For $a, a \times a = 0$.
5. For vectors a and $b, a \times b = -b \times a$, i.e. cross product of two vectors is not commutative.
6. For a, b and $c, a \times (b \times c) \neq (a \times b) \times c$, in general.
7. Distributive property: For vectors a, b and $c, a \times (b + c) = a \times b + a \times c$
8. For vectors a and $b, a \times b = 0 \Leftrightarrow a = 0, b = 0$ or $a \parallel b$.
9. If we want to show that two non-zero vectors a and b are parallel, then we should show that $a \times b = 0$.
10. Geometrically, $|a \times b|$ represents the area of a parallelogram whose adjacent sides are along a and b .
11. Area of a triangle whose sides are along a and b is given by $1/2 |a \times b|$.
12. Area of a parallelogram whose diagonals are along d_1 and d_2 is given by $1/2 |d_1 \times d_2|$.
13. If i, j, k are vectors along x, y and z -axes respectively, then
 $i \times i = 0, j \times j = 0, k \times k = 0, i \times j = k;$
 $j \times k = i; k \times i = j.$
14. For a scalar $\lambda, \lambda(a \times b) = (\lambda a \times b) = (a \times \lambda b)$, where a and b are given vectors.

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[CLASS XII]

15. If $a = a_1i + b_1j + c_1k$; $b = a_2i + b_2j + c_2k$, then $a \times b =$

$$\begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

16. Unit vector \hat{n} perpendicular to a and b is $\hat{n} = a \times b / |a \times b|$

(i) If we have to find unit vectors perpendicular to a and b , then we take $\pm \hat{n}$.

(ii) If we have to find a vector of magnitude k units and perpendicular to vectors a and b , then we find $k \hat{n}$, i.e. $k (a \times b / |a \times b|)$

(iii) If we have to find vectors of magnitude k units and perpendicular to vectors a and b , then we find $\pm k \hat{n}$, i.e. $\pm k (a \times b / |a \times b|)$

❖ SOLVED EXAMPLES

Very Short (Objective Type) / Short Answer Type

- If for non zero vectors a and b , $|a \times b|$ is a unit vector and $|a| = |b| = \sqrt{2}$, then angle θ between vectors a and b is
(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/6$ (d) $-\pi/2$
- The area of a parallelogram whose one diagonal is $2i + j - 2k$ and one side is $3i + j - k$ is
(a) $i - 4j - k$ (b) $3\sqrt{2}$ sq units
(c) $6\sqrt{2}$ sq units (d) 6 sq units
- Find the angle between two vectors a and b with magnitudes 1 and 2 respectively and when $|a \times b| = \sqrt{3}$.
- Find λ , if $(2i + 6j + 14k) \times (i - \lambda j + 7k) = 0$.
- If $a \times b = b \times c$ and $a \times c = b \times a$, prove that $a - b$ is parallel to $b - c$, provided $a = d$ and $b = c$.
- Show that the area of the parallelogram having diagonals $(3i - j - 2k)$ and $(i - 3j - 4k)$ is $5\sqrt{3}$ sq units.
- Write the value of $(i \times j) \cdot k + j$. [AI 2012]
- Write a unit vector perpendicular to both the vectors $a = i + j + k$ and $b = i + j$. [AI 2015]
- If $a = i - j - k$ and $b = j - k$ and $a \times c = b$ and $a \cdot c = 3$.

10. Find a unit vector perpendicular to each of the vectors $a + b$ and $a - b$, where $a = 3i + 2j + 2k$ and $b = i + 2j - 2k$. [NCERT: Delhi 2001]
11. a, b, c are unit vectors, suppose $a \cdot b = a \cdot c = 0$ and angle between b and c is θ . Prove that $\vec{a} = \pm 2(b \times c)$.
[CBSE Sample paper 2016]
12. If for three non-zero vectors a, b and c , $a \cdot b = a \cdot c$ and $a \times b = a \times c$, then show that $b = c$.
13. Using vectors, find the area of the triangle with vertices $A(1, 1, 2)$, $B(2, 3, 5)$ and $C(1, 5, 5)$. [AI 2011]
14. Find a vector of magnitude 6, perpendicular to each of the vectors $a + b$ and $a - b$, where $a = i + j + k$ and $b = i + 2j + 3k$.
[Foreign 2013]
15. If $r = xi + yj + zk$, find $(r \times i) \cdot (r \times j) + xy$.
16. Show that the points A, B, C with position vectors $2i - j + k, i - 3j - 5k$ and $3i - 4j - 4k$ respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle. [AI 2017]
17. Let $a = 4i + 5j - k, b = i - 4j + 5k$ and $c = 3i + j - k$. Find a vector d which is perpendicular to both c and b and $d \cdot a = 21$.

❖ **Very Short (Objective Type) / Short Answer Type**

1. If $|a| = 8, |b| = 3$ and $|a \cdot b| = 12\sqrt{3}$ then the value of $|a \times b|$ is
(a) 12 (b) $12\sqrt{3}$ (c) 6 (d) $4\sqrt{3}$
2. Vectors a and b are such that $|a| = \sqrt{3}, |b| = 2/3$ and $(a \times b)$ is a unit vector. Write the angle between a and b .
3. Find the area of a parallelogram whose adjacent sides are represented by the vectors $2i - 3k$ and $4j + 2k$.
[Foreign 2015]
4. If a and b are two vectors such that $|a| = |a \times b|$. Then what is the angle between a and b ?
5. Write the value of the area of the parallelogram determined by the vectors $2i$ and $3j$.
[Foreign 2012]

❖ Long Answer I / Long Answer II Type

- Let $a = i + 4j + 2k$, $b = 3i - 2j + 7k$ and $c = -2i - j + 4k$. Find a vector d which is perpendicular to both a and b and $c \cdot d = 18$.
- If $a \times b = b \times c \neq 0$, show that $a + c = mb$, where m is a scalar.
- Find a vector whose magnitude is 3 units and which is perpendicular to the following two vectors:
 $a = 3i + j - 4k$; $b = 6i + 5j - 2k$.

SCALAR TRIPLE PRODUCT

- Scalar triple product: $a \cdot (b \times c)$ or $(a \times b) \cdot c$.
 For vectors a, b, c , scalar triple product is given by $a \cdot (b \times c)$ or $(a \times b) \cdot c$.
 Scalar triple product is also denoted by $[a \ b \ c]$.
- For vectors $a = a_1i + b_1j + c_1k$, $b = a_2i + b_2j + c_2k$ and $c = a_3i + b_3j + c_3k$

$$[a \ b \ c] = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$
- $[a \ b \ c] = - [b \ c \ a] = - [c \ a \ b]$.
- $[a \ b \ c] = - [c \ a \ b]$.
- $[a \ b \ b] = 0$; $[a \ a \ b] = 0$.
- If $[a \ b \ c] = 0$, then vectors a, b, c are coplanar and vice versa.
- If vectors $a = a_1i + b_1j + c_1k$, $b = a_2i + b_2j + c_2k$ and $c = a_3i + b_3j + c_3k$ are coplanar,
 then $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$
- $[a \ b \ c]$ geometrically represents volume of a parallelepiped whose adjacent sides are along a, b, c .
- a, b, c are three non zero vectors such that $a \times b = c$, $b \times c = a$. Prove that a, b, c are mutually at right angles and $|b| = 1$, $|c| = |a|$. [HOTS]
- If $a = i + j + k$ and $b = j - k$, find a vector c such that $a \times c = b$ and $a \cdot c = 3$. [NCERT Exemplar]
- If $a + b + c = 0$, show that $a \times b = b \times c = c \times a$. Also, interpret the result geometrically. [NCERT Exemplar]
- Find a unit vector perpendicular to both of the vectors $a + b$ and $a - b$ where $a = i + j + k$, $b = i + 2j + 3k$.

LOYAL EDUCATION MATHEMATICS

[Foreign 2014]

Result Oriented

(DAILY PRACTICE PAPER)

[CLASS XII]

13. If a, b, c are position vectors of vertices A, B, C of a triangle ABC , show that area of a triangle is $\frac{1}{2} |a \times b + b \times c + c \times a|$.
14. Find the area of triangle OAB , where $OA = 3i - j + k$ and $OB = -2i + j - 3k$.
15. If $a \times b = a \times c \neq 0$, show that $b = c + \lambda a$, for some real number λ .
16. For any two vectors a and b , prove that $(a \times b)^2 = |a|^2 |b|^2 - (a \cdot b)^2$.
17. Define vector product $a \times b$ of two vectors a and b . If $|a| = 2, |b| = 5$ and $|a \times b| = 8$, find the value of $a \cdot b$. [HOTS]

1. If for three non zero vectors $a, b, c, |a \cdot b \cdot c| = 0$, then vectors a, b, c are _____.
2. Find λ , if the vectors $a = i + 3j + k, b = 2i - j - k$ and $c = \lambda j + 3k$ are coplanar. [Delhi 2015]
3. Find λ , so that the vectors $2i - j + k, i + 2j - 3k$ and $3i + \lambda j + 5k$ are coplanar.

❖ Long Answer I / Long Answer II Type

4. Find the volume of a parallelepiped whose continuous edges are represented by vectors $a = 2i - 3j + k, b = i - j + 2k$ and $c = 2i + j - k$.
5. Let a, b, c , be three non-zero vectors. If $a \cdot (b \times c) = 0$ where b, c are not parallel, then prove that $a = \lambda b + \mu c$, where λ and μ are scalar constants.
6. Find x , such that the points $A(3, 2, 1), B(4, x, 5), C(4, 2, -2)$ and $D(6, 5, -1)$ are coplanar.
7. Show that the vectors a, b, c are coplanar/non-coplanar if $a + b, b + c, c + a$ are coplanar / non-coplanar. [NCERT: Delhi 2014]
8. Show that the four points A, B, C and D with position vectors $4i + 5j + k, -j - k, 3i + 9j + 4k$ and $4(-i + j + k)$ respectively are coplanar. [AI 2014]
9. If $a = i - 2j + 3k, b = 2i + 3j - 5k$, then find $a \times b$. Verify that a and $a \times b$ are perpendicular to each other.

❖ PRACTICE QUESTIONS

* Short (Objective Type) / Short Answer Type

- 1) If vectors $i + j - 3k, 2i + j - \lambda k, 5i + 2j + 3k$ are coplanar, then value of λ is
- (a) 4 (b) 0 (c) -3 (d) 2

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[CLASS XII]

- 2) Find $a \cdot (b \times c)$, if $a = 2i + j + 3k$, $b = -i + 2j + k$ and $c = 3i + j + 2k$.
[AI 2014]
- 3) If $a = i - j + k$ and $b = 3i + 2j + 4k$, find $[a \ b]$.
- 4) Position vectors of points A and B are $a + b$ and $2a - b$. Then AB equal to
(a) $3a$ (b) $-a + 2b$ (c) $a - 2b$ (d) none of these
- 5) Given vector a , then $-2a$ is a vector whose
(a) magnitude is twice that of a and direction is same as that of a
(b) magnitude is twice that of a and direction is opposite to that of a
(c) magnitude is same as that of a and direction is opposite to that of a
(d) none of these
- ❖ **Long Answer I / Long Answer II Type**
- 6) If any three vectors a, b, c are coplanar, show that the vectors $a + b, b + c$ and $c + a$ are also coplanar.
[DoE; Delhi 2016]
- 7) Using vectors show that the points $A(-1, 4, -3)$, $B(3, 2, -5)$, $C(-3, 8, -5)$ and $D(-3, 2, 1)$ are coplanar.
- 8) Let $a = i + j + k$, $b = i$ and $c = c_1i + c_2j + c_3k$, If $c_2 = -1$ and $c_1 = 1$, show that no value of c_3 can make a, b, c coplanar.
[NCERT; Delhi 2017]
- 9) Show that the vectors a, b, c are coplanar if and only if $a + b, b + c$ and $c + a$ are coplanar.
[Foreign 2014]
